



# Small Volume Prover Uncertainty Analysis FMD-035



P: +1.602-233-9885 | [fmd-sales@idexcorp.com](mailto:fmd-sales@idexcorp.com) | [www.flowmd.com](http://www.flowmd.com)

"Always Improving Liquid Measurement Technology"

■ SERVICE, EXPERIENCE, INNOVATION & EXCELLENCE



## INTRODUCTION

Following are the three main elements of any measurement that describe how closely it represents reality:

1. Accuracy
2. Repeatability
3. Uncertainty
  - a) Systematic
  - b) Random

Accuracy describes how close the measurement is to the true answer. Repeatability describes how closely the same answer will result if the measurement is taken many times over. Uncertainty describes the tolerance or resolution of measurements taken with an instrument. The total uncertainty of an instrument consists of both systematic uncertainty and random uncertainty. The systematic uncertainty is determined both by experience and comparison to other systems. Random uncertainty must be determined by experiment on a specific piece of equipment. Systematic uncertainties in the FLOW MANAGEMENT DEVICES flow prover result from the prover volume calibration, as well as from uncertainties associated with operating the system to prove flow meters. While both prover volume calibration and operation incur systematic uncertainties and random uncertainties, the systematic uncertainty of the prover during operation must include the random uncertainties of the gravimetric calibration. Random uncertainties during operation will vary greatly depending upon flow rate, fluid, and the type and size of flow meter being proved. These random uncertainties, under a particular set of operating conditions, add to the uncertainty.

This analysis has been performed in two parts:

Part 1. Uncertainty of calibration by gravimetric method

Part 2. Systematic Uncertainty of flow meter calibration

**Note: The symbols used in the following equations have been used for simplicity, and are not necessarily the same symbols used in the equations in API 4.9.4.**

## UNCERTAINTY OF CALIBRATION BY GRAVIMETRIC METHOD

### Volumetric Calibration Details

For this analysis, an FMD-035 prover is used with a cylinder bore of 17.002 with and a displaced volume of 94.6 liters between optical detectors. An automatic solenoid operated water draw apparatus is used for dispensing the water into the catch container on the electronic weigh scale. De-ionized water is collected between the two optical detectors, providing an apparent mass of approximately 94664 grams. With all measurements and calculations conforming to the appropriate API Manuals of Measurement Standards, an equivalent volume of 94630.546 milliliters is calculated for the prover from the mass measurement. Five draws were taken and averaged to determine a base volume constant for the prover.

### System Equation for Calibration by Gravimetric Method

The equation for the mass of water,  $M_t$ , in grams collected in the weigh tank is given by:

$$M_t = W_a * \{1 + \rho_a[(1/\rho_w) - (1/7.84)]\}$$

Where:  $W_a$  = Weight of water collected in the weigh tank, in grams  
 $\rho_a$  = Density of air during the weighing, in grams per cc  
7.84 = Density of the mass standards used for calibration of the weigh scales  
(If mass standard density other than 7.84, use correct density in calculation)  
 $\rho_w$  = Density of water in weigh tank in grams per cc

Equation for the base volume,  $V_b$ , between the detectors at reference conditions is given by:

$$V_b = M_t / (\rho_p * C_{psp} * C_{tsp})$$

Where:  $\rho_p$  = Density water in the prover at actual temperature and pressure during calibration  
 $C_{psp}$  = Correction factors for expansion of prover due to pressure during calibration  
 $C_{tsp}$  = Correction factors for expansion of prover due to temperature during calibration

Note: Formulas, coefficients, and explanations of  $C_{psp}$  and  $C_{tsp}$  can be found in API 12.2.1

Uncertainties in the terms of these equations comprise the systematic uncertainty of the gravimetric calibration process.

Note: In actual water draw practice, all procedures and operations are performed in strict accordance with all applicable standards, such as API Chapters 4.9.4, 12.1, 12.2.4, 11.4.1, 4.2, 14.6, and complies with the fourth draft revision of ISO/WD 7278-5 (not an ISO standard). The information shown in this paper is for analytic purposes only. For more complete volumetric calibration information on FLOW MANAGEMENT DEVICES flow provers, please refer to applicable API and ISO documents, as well as the FLOW MANAGEMENT DEVICES Flow Prover Operation Manual.

### Sources of Uncertainty in the equation

$W_a$	The weight of the water collected = +/- 0.00244%, due to the uncertainty of the electronic balance used, and the uncertainty of the test weights used to calibrate the balance.
$\rho_w$	The density of the water = +/- 0.0008%, due to the Patterson Morris density equation with temperature measurement accuracy of 0.1 degree F of the water drawn from the cylinder.
$\rho_a$	The density of the air around the scale, subtracted from the density of water to correct for the buoyancy of water in air = +/- 0.173%, due to uncertainty of barometric pressure (+/- 0.05 in Hg) and temperature (+/- 0.1 degree C) in the calculation of air density.
$T_d$	Temperature measurement of volume detector mount during calibration with temperature measurement accuracy of 0.1 degree F.
$T_p$	Temperature measurement of prover flow tube during calibration. Utilizing a temperature measurement accuracy of 0.1 degree F.
$P_p$	Pressure measurement of prover flow tube during calibration. Utilizing a measurement accuracy of 0.5 psi.

### Summary of Systematic Uncertainties of Calibration

Variable	Nominal Value	Error%	Sensitivity	Sensitivity * Error %
$\rho_w$	0.997239 g/cm <sup>3</sup>	+/- 0.0008	1	+/- 0.0008
$\rho_a$	0.001119 g/cra <sup>3</sup>	-/+ 0.173	0.0012	+/- 0.0002
$W_a$	94630.546 ml	+/- 0.00261	1	+/- 0.00261
$T_d$	59.9 deg. F	+/- 0.0001	1	+/- 0.0001
$T_p$	57.4 deg. F	+/- 0.0002	1	+/- 0.0002
$P_p$	20.0 psi	+/- 0.0001	1	+/- 0.0001

NOTE 1: Sensitivity indicates the effect a term will have on the prover constant.

NOTE 2: In actual practice, air buoyancy calculations are taken from API 14.6.14.2, and are based on altitude. With insignificant error, this practice simplifies the air density calculation. An uncertainty of up to +/- 0.05 in Hg shown in the above equation could occur due to maximum barometric changes due to local weather conditions, but can be seen from above summary, is very insignificant.

The combined systematic uncertainty is the square root of the sum of the squares of the individual uncertainties.

$$\begin{aligned} \text{Total systematic uncertainty} &= +/- \sqrt{(\rho_p^2 + \rho_a^2 + W_a^2 + T_d^2 + T_p^2 + P_p^2)} \\ &= +/- 0.00275\% \end{aligned}$$

## Evaluation of Random Uncertainties of Gravimetric Volumetric Calibration

Random uncertainties for calibration in this analysis are derived from the water draw data for the selected prover. Since these uncertainties are part of the base volume constant, they contribute to the systematic uncertainty of the FLOW MANAGEMENT DEVICES flow prover. Sources of random uncertainty are:

Start/Stop variations in electric control valves, random weighing errors, and random errors in temperature and pressure measurement.

Every flow prover water draw will have slightly different random uncertainties. For this analysis, a FLOW MANAGEMENT DEVICES model FMD-035 serial number 000005 was used to determine the estimated limits of random uncertainty. This is accomplished by taking the mean and standard deviation of the several draws taken during water draw calibration. The limits of uncertainty are calculated to the 95% confidence level. This is accomplished by taking the mean and standard deviation of the several draws taken during water draw calibration. The limits of uncertainty are calculated to the 95% confidence level. The average weight of the draws taken was 94464.2 gm, and the Standard Deviation of the draws was 1.9955 gm (0.00157%). The estimated limits of random uncertainty at the 95% confidence level (CL) are:

$$\begin{aligned} \text{CL} &= \pm t * s \\ &= \pm 2.365 * 0.00158\% \\ &= \pm 0.00371 \% \end{aligned}$$

Where:  $t$  = Student's  $t$  for 95% confidence and six degrees of freedom.

$s$  = Sample standard deviation.

This means that we can have 95% confidence that all draws will fall within this  $\pm 0.00261\%$  band of draw volumes.

## Total Estimated Uncertainty of Calibration

To determine the worse case total uncertainty of the volume constant calibration, we add the systematic uncertainty and the random uncertainty at the 95% level. The result is a figure that represents the estimated limits of variation in the prover's volume constant.

$$\begin{aligned} \text{Total Estimated Uncertainty} &= \pm (0.00275\% + 0.00371 \%) \\ &= \pm 0.00646\% \end{aligned}$$

## SYSTEMATIC UNCERTAINTIES OF FLOWMETER CALIBRATION

During actual proving of flow meters, several additional uncertainties add to the basic uncertainty of water draw calibration. For this analysis, we begin with the system equation for operation.

$$\text{Volume K} = C_m * (T_v / V_b) * V_b * \text{CCF}$$

Where:

$C_m$  = Number of whole flow meter pulses.

$T_v$  = Volume time between optical switches.

$T_m$  = Time for the whole flow meter pulses.

$V_b$  = Base volume of the prover.

CCF = Combined Correction factors for the prover base volume.

In this equation, the meter pulses multiplied by the ratio of volume time to meter time, is multiplied by the corrected prover volume ( $V_b$  corrected to ambient conditions).

### Sources of Uncertainty in the System Equation

Number of whole flow meter pulses. Since only whole meter pulses are read, and a whole count error would be detected by the system, in this analysis pulse factors are considered to have zero error. (Refer to API Petroleum Measurement Standard Chapter 4 Section 2 for analysis calculations for uneven pulse train flow meters) Error in Volume Time between optical switches =  $\pm 0.0001\%$  based on calibration of the 1 Mhz crystal oscillator. Error in Time for the whole flow meter pulses =  $\pm 0.0001\%$  based on calibration of the 1 Mhz crystal oscillator. Error in Base volume of the prover =  $+ 0.00713\%$  from the calibration analysis. Error in Correction factors for the prover base volume =  $+ 0.0031\%$ . For this presentation, a detailed analysis of these correction factors is omitted. The error contribution of these factors is, however, included in the summary table of uncertainties below.

### Summary of Systematic Uncertainties

Nominal Sensitivity			
Variable Value	Error %	Sensitivity	* Error %
$e_{T_v}$ Volume time 1 sec	$\pm 0.0001\%$	1	$\pm 0.0001\%$
$e_{T_m}$ Meter pulse time 1 sec	$\pm 0.0001\%$	1	$\pm 0.0001\%$
$e_{V_b}$ Base volume 94630.546 ml	$\pm 0.00646\%$	1	$\pm 0.00643\%$
eCCF Corrections			
Switch bar temperature 1.000353	$\pm 0.00038\%$	1	$\pm 0.00038\%$
Tube temperature 0.998506	$\pm 0.0027\%$	1	$\pm 0.0027\%$
Tube pressure 0.99995	$\pm 0.00005\%$	1	$\pm 0.00005\%$

NOTE: Sensitivity indicates the effect the term will have on the Volume K of the flow meter under test. The combined systematic uncertainty is taken as the square root of the sum of the squares of the individual uncertainties.

$$\begin{aligned} \text{Total systematic uncertainty} &= \pm \sqrt{(e_{T_v}^2 + e_{T_m}^2 + e_{V_b}^2 + e_{CCF}^2)} \\ &= \pm 0.00765 \% \end{aligned}$$

### What This Means in Practical Terms

In analyzing the the chain of uncertainties in this system, we start with the tolerance of the test weights used to calibrate the scales, followed by uncertainty in measuring temperatures and pressures to calculate volume from mass. These comprise the uncertainty of the FLOW MANAGEMENT DEVICES flow prover basic calibration. When proving flow meters, the uncertainty of measuring the passage of time, and tolerances in correction factors that relate all readings to standard pressure and temperature, complete the chain. All of the uncertainties combined amount to a worst case systematic uncertainty of less than one part in fifteen thousand (.00765%). This analysis covers possible errors in the prover. An actual meter provings will show additional errors that are contributed by the flow meter being proven. For a more detailed description of double chronometry timing, the effects of non-uniform flow meter pulse spacing, flow variations, and other effects on flow meter proving results, refer to API 4.2..